

## CONDITIONALS IN INTERACTION

ABSTRACT. There are several issues with the standard approach to the relationship between conditionals and assertions, particularly when the antecedent of a conditional is (or may be) false. One prominent alternative is to say that conditionals do not express propositions, but rather make *conditional* assertions that may generate categorical assertions of the consequent in certain circumstances. However, this view has consequences that jar with standard interpretations of the relationship between proofs and assertion. Here, I analyse this relationship, and say that, on at least one understanding of proof, conditional assertions may reflect the dynamics of proving, which (sometimes) generate categorical assertions. In particular, when we think about the relationship between assertion and proof as rooted in a dialogical approach to both, the distinction between conditional and categorical assertions is quite natural.

### 1. ISSUES WITH CONDITIONALS

There are well-worn issues with the way in which conditionals are supposed to be understood, particularly when they have false antecedents. The standard treatment of conditionals tells us that  $\alpha \rightarrow \beta$  is true whenever  $\alpha$  is false.<sup>1</sup> But, in English, when we say that “ $\alpha$ , therefore  $\beta$ ”, or “if  $\alpha$ ,  $\beta$ ”, if  $\alpha$  were shown to be false, it does not seem correct to say that the conditionals are true. According to many critics [e.g. 14, 32, 33], this (amongst other issues) suggests that asserting a conditional does not express a “conditional” proposition. That is to say, whatever is distinctive about uttering a conditional statement is not that it is an assertion that expresses a distinctive kind of propositional content.<sup>2</sup> Rather, such an assertion is to be thought of as a *conditional* assertion of the consequent, where the condition is the antecedent. This, conditional, assertion, then, is a distinctive speech act, as Stalnaker [38] points out, not a standard speech act (assertion) with distinctive content.

There are different ways of thinking about conditional assertions. For example, some proponents have thought that whenever the antecedent is false, the condition

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<sup>1</sup>See the excellent discussion in [17, ch.7] and [36].

<sup>2</sup>This distinction does not cohere with distinction between the between a conditional assertion (if  $\alpha$ , then I assert that  $\beta$ ) and the assertion of a conditional (I assert that, if  $\alpha$ , then  $\beta$ ), which I do not discuss in the following, noting only that there may be interesting complications with assertions of these forms.

upon which the assertion of the consequent is made does not hold, and so no assertion is made. Quine puts this as follows:

An affirmation of the form If  $\alpha$  then  $\beta$  is commonly felt less as an affirmation of a conditional than as a conditional affirmation of the consequent. If, after we have made such an affirmation, the antecedent turns out true, then we consider ourselves committed to the consequent, and are ready to acknowledge error if it proves false. If on the other hand the other hand the antecedent turns out to have been false, our conditional affirmation is as if it had never been made. [32, §3]

Then, as Humberstone [17] suggests, after Ramsey [33], to assert a conditional is not to be thought of as asserting a conditional proposition, but to make a conditional assertion of the consequent: ‘If the latter condition is not satisfied (i.e., if the antecedent is false), then it is as though no assertion had been made. The parallel is with conditional bets, which are void in that no money changes hands unless the condition they are conditional upon obtains’ (p.938). This is also captured in the infamous statement by Ramsey:

If two people are arguing “If  $p$  will  $q$ ?”, and are both in doubt as to  $p$ , they are adding  $p$  hypothetically to their stock of knowledge and arguing on that basis about  $q$  [...] If either party believes not  $p$  for certain, the question ceases to mean anything to him except as a question about what follows from certain laws or hypotheses.[33, p.247]

Whilst this approach does seem to capture something correct about the treatment of conditionals with false antecedents, it is, nonetheless, tricky to explain how these conditional assertions that may fail to assert anything are supposed to interact with standard approaches to speech-acts, particularly in terms of how they interact with propositional content.<sup>3</sup> For example, take a conditional like the following: “If you press that switch, there will be an explosion”. When stated by one agent to another, even if the antecedent does not hold (the hearer does not press the switch), it does not seem that nothing has been said, since the hearer has learned some sort of reason to think that, should the antecedent hold, then the consequence will be an explosion!<sup>4</sup> This suggests that conditional assertions are not “empty”, but that they are distinct from making a categorical assertion of the consequent. As Edgington [13] puts it: ‘My hearer understands that if she presses [the switch], my assertion of the consequent has categorical force; and given that she takes

<sup>3</sup>See the discussion in [38] for some suggestions.

<sup>4</sup>This follows Edgington’s [13] argument.

me to be trustworthy and reliable, if it does acquire categorical force, it is much more likely to be true than false. So she too acquires reason to think that there will be an explosion if she presses it, and hence a reason not to press it' (p.178). An additional, though less discussed issue for the approach, is that it does not cohere well with standard accounts of deduction. It is difficult to know whether an argument involving conditionals as premises or conclusion is valid. Take the following argument:

$$\frac{[\alpha] \quad \beta}{\alpha \rightarrow \beta} (\rightarrow\text{-I})$$

where  $[\alpha]$  indicates that  $\alpha$  is an assumption. Now, say that  $\alpha$  is false, then, the conditional assertion codified by the inference step from  $[\alpha]$  to  $\beta$  expresses nothing, and, so presumably no inference is made. So, the movement from  $[\alpha]$  to  $\alpha \rightarrow \beta$  is disconnected, leaving a “gap” in the deduction.

What the standard approach to conditionals, and the conditional assertion approach share is the idea that categorical assertions are the “industry standard”, as it were, with conditional assertions (if allowed) to be explained in terms of them. As a result, there is an “all or nothing” status awarded to assertions, so, inevitably, conditional assertions are thought to be empty if they do not become categorical. To the contrary, I here pursue the idea that conditional assertions may be better understood in terms of the *dynamics* of logical reasoning inside dialogical situations. This is to take conditional assertions as the norm, with categorical assertions generated through dialogical interaction, and in certain circumstances. That is to say, we can see conditional assertion in terms of the dynamic process of reasoning, with categorical assertion as the objects (sometimes) produced by that process.<sup>5</sup> To get there, I begin in §2 by discussing the usual understanding of the relationship between proof and categorical assertion, and show that it rules out the conditional assertions approach altogether. In §2.1, I draw attention to a divergent view of proofs, which emphasises the activity of proving, and suggests that there might be a place for conditional assertions alongside categorical assertions. I begin §3 by drawing attention to an analogous approach to assertions in terms of the social dynamics of commitment, before, in §3.1, showing that, by placing both assertions and proofs in a social context, a natural approach to conditional assertion emerges.

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<sup>5</sup>I will not consider model-theory in what follows, but it is worth saying that I do not think an approach making use of a partial interpretation function will deliver the correct results. I develop an *inferentialist* semantics for interactions that is consistent with the below account in [40].

## 2. DEDUCTION AND CATEGORICAL ASSERTION

Let us consider natural deduction in intuitionistic form, primarily because this logic does not seem to force the categorical assertion view upon us in the same way as classical logic, where  $\alpha \rightarrow \beta$  is equivalent with  $\neg\alpha \vee \beta$ . In fact, some of the ingredients of the conditional assertion view are already apparent in the Brouwer-Heyting-Kolmogorov (BHK) interpretation of a conditional: A proof for  $\alpha \rightarrow \beta$  is a function  $f$  which, to each proof  $a$  of  $\alpha$  provides a proof  $f(a)$  of  $\beta$ . But, whilst this suggests that such an account is amenable to conditional assertion, this turns out not to be the case, at least according to the most prominent interpretation of the validity of proofs in natural deduction.

According to Prawitz and Dummett, validity is definitional of what a proof is, and its validity is relative to a formal entailment structure, so to ask whether or not a proof is valid is nonsensical.<sup>6</sup> Prawitz does, however, consider “closed” and “open” arguments, showing how to define validity for these such that a valid closed argument is equivalent with a proof. An open argument is just an argument that involves undischarged assumptions, or unbound variables. In contrast, a closed argument has no assumptions, and is valid just in case it is *either* canonical, so that it ends with an instance of an introduction rule (in natural deduction calculus), *or* it can be reduced to a canonical argument for the conclusion.<sup>7</sup> Then, according to Prawitz, it is also possible to say that a closed argument is valid iff it can be identified with a proof, and an open argument is valid if it can be reduced to a closed canonical argument:

**Definition 1.** (Prawitz-Dummett definition of validity): An argument  $A$  is valid whenever:

- $A$  is closed and canonical;
- or  $A$  is closed and reduces to a canonical argument;
- or  $A$  is open and reduces to a closed canonical argument;

In other words, the emphasis is on closed and canonical arguments in a deductive system, which allows for the reduction of non-canonical to canonical arguments, and also ensures that for a canonical argument to be valid requires its immediate subproofs to be valid. The justification of open arguments relies upon this prior

<sup>6</sup>The following is based upon the accounts in [10, 31, 41].

<sup>7</sup>It is reductions that ensure that a derivation can be normalized, since successive reduction procedures ensure that any “roundabouts” in the derivation can be eliminated. Dummett, in [10, p.254], calls the fact that every closed derivation in an intuitionistic entailment structure can be reduced to a canonical derivation, the “fundamental assumption”. Whenever the introduction and elimination are in “harmony”, this ensures that if the conclusion of an introduction rule is also the major premise of an elimination rule (at some point in a derivation), then it is possible to reduce that derivation to a derivation with the same premises and conclusion, without the “detour” through those steps.

notion, by taking the open argument and replacing all open assumptions with closed proofs (or open variables with closed terms). So, in general, a proof of  $\beta$  under the assumption  $\alpha$  is valid whenever it is possible to replace the assumption  $\alpha$  with a (valid) closed proof of  $\alpha$ .<sup>8</sup>

This approach to the validity of arguments accords with an approach to assertions that privileges categorical assertions, and, in fact, is at odds with any appeal to conditional assertion. This is, in part due to the interpretation of proofs as providing the *objectively correct* conditions for an assertion:

[...] in the general case, we have to consider as primary, in determining the content of an assertion, not the speaker's personal entitlement to make the assertion, but the condition for its objective correctness.[9, p.120]

These objective correctness conditions for sentences involving logical constants are just formally derivable proofs (or valid arguments), which must be stable, objective, and timeless.<sup>9</sup> As such, this coheres with the, now, fairly dominant understanding of proofs as objective, a view found in Prawitz [31] and Dummett [e.g. 12], which requires only that there exist an effectively decidable possible proof for a statement, and not an actual proof carried out by an agent.<sup>10</sup> Insofar as proofs are understood in objective terms, the *act* of proving is reduced to a kind of ratification, where a proof itself is unaffected by our interaction with it. In order for a sentence to be asserted, the kind of evidence that is required is a closed proof. Open proofs are assertible, only insofar as they can be reduced to closed proofs. That is, if we have an open proof of  $\beta$  from  $\alpha$ , then  $\beta$  could not be asserted unless we have some evidence that  $\alpha$  holds also. It is a small step to see that, on this view, the only kind of assertion of interest is categorical, since there is no “room for manoeuvre”: either a proof exists or it doesn't, and whether or not a proof exists is what determines the correctness of assertions.

This is a general feature of this approach to the relationship between proof and assertion, but, unsurprisingly, this becomes most clear in the case of the conditional, by which it is possible transform an open proof into a closed proof. Take, for example, a standard derivation introducing a conditional:

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<sup>8</sup>As [34] puts it, according to Prawitz, '[a]n argument is valid if either it reduces to a non-logical justification of an atomic sentence, or it reduces to an argument whose last inference is an introduction inference and whose immediate subarguments are valid', (p.7).

<sup>9</sup>Furthermore, this objectively true notion of the proof of a statement is equivalent with its truth, according to Dummett [9].

<sup>10</sup>This may still be considered “anti-realist” from the point of view of an agent's epistemic access to proofs, but proofs may be understood to be agency-independent insofar as they are independent from an agent's actual proving-activities. See the discussion in [7].

$$\frac{[\alpha]}{\alpha \rightarrow \beta} (\rightarrow\text{-I})$$

Here, we have a proof of  $\alpha \rightarrow \beta$ , which, given a proof of  $\beta$ , no longer depends upon  $\alpha$  as assumption. That is to say, it is closed and canonical, so it is a valid proof, by definition, which introduces the conditional statement, whether or not  $\alpha$  holds.<sup>11</sup> By the connection between assertion conditions and proofs, we also know that, in this case,  $\alpha \rightarrow \beta$  must be categorically assertible. This becomes even clearer when we consider negation, defined (intuitionistically) as  $\neg\alpha =: \alpha \rightarrow \perp$  (where  $\perp$  expresses a constantly false proposition), since, if we say that  $\alpha \rightarrow \perp$  is assertible on condition that  $\alpha$  holds, whilst, definitionally, neither  $\perp$  nor  $\alpha$  can not hold, then  $\neg\alpha$  could never be asserted.<sup>12</sup> Conditional assertions are ruled out, by definition.

**2.1. Conditional dynamics.** There is a different view of proofs, which coheres somewhat better with conditional assertions, and takes agents' proving activity to be central. This is to think of proofs as *acts*, rather than as objective entities. For example, the objective approach to proofs (as pointed out in [e.g. 23, p.84-5]) seems to require commitment to an objective realm of propositions, leading to an "inert platonism of proofs" [7]:

[f]ar from being deduced or extracted by ourselves, the consequences of an hypothesis follow from it by themselves, or rather in virtue of the existence of certain objects that it is none of our responsibility to conceive, or to make up, but only to discern [...] By identifying proofs with sequences of formulas or, more generally, with objects that are independent from us, one almost unavoidably reduces the activity of justification to a scanning and control process that requires no cognitive or physical particular resource . . . . [6]

Instead, according to Dubucs and Marion [7] we should think of proofs as "acts", rather than "objects":

We propose that one distinguishes between two different notions of proof, namely those of proof as 'object' and as 'act'. According to the first conception, a proof is something like an assemblage of strings of symbols satisfying such and such property. From the second, more dynamic, conception, a proof is a process whose result may be represented or described by means of linguistic symbols.

<sup>11</sup>Note that in the standard Brouwer-Heyting-Kolmogorov semantics, categorical assertions are privileged, though Kolmogorov's own interpretation in terms of "problems" bears some resemblance to the below account. For discussion, see [5].

<sup>12</sup>This is discussed in [24].

On this view, a proof is understood as an *act* that is undertaken by agents, allowing that there is some sort of *dynamics* to proofs insofar as they are carried out in time. That is, we may take the act of proving seriously insofar as logic is taken not just to deal with propositional and objective, but with the *actions* of reasons themselves. The process of “proving”, then, is more like a process of reasoning that is not required to live up to objective correctness conditions on assertions, which may be thought of as *generating* proof-objects. The latter is something like an *actual* proof, which for a conditional,  $\alpha \rightarrow \beta$ , is just a function that maps *actual* proofs of the  $\alpha$  into *actual* proofs of  $\beta$ . Such a function can not map on assumptions, according to this view, since then, the function could not be a map at all: ‘as long as no proof of  $\alpha$  is known, [the function]  $f$  has nothing to map. So we can still define  $f$  as the constant function which, once a proof  $\pi$  of  $\alpha$  is known, maps every proof of  $\alpha$  into the proof of  $\beta$ ’ [23, p.91].<sup>13</sup> In this regard, of particular note is the distinction made by Martino and Usberti [23] between what we may call a *hypothetical* function  $f_H$  that would come into effect once we have a proof of the antecedent to hand, and the *actual* function  $f_A$  which maps the antecedent into the consequent when we have the proof of the antecedent. The latter is *actual* since the function is only then an actual map that has come into effect given that a proof of the antecedent of the conditional is available. Since the conditional is intended to “write-into” the object language the relationship defined by the turnstile, and by the functional definition of a proof given at the start of this section, we can generalise this distinction as follows. In general, a proof of  $\alpha \vdash \beta$  is just a function that takes  $\alpha$ , and maps it into  $\beta$ , hypothetical just in case the required evidence for  $\alpha$  is not to hand, and actual just in case there is such evidence. As such, we now have a distinction between a hypothetical proof (in which we assume  $\alpha$ ), and a valid proof, in which case a proof of  $\alpha$  is also given.<sup>14</sup> This, moreover, is a distinction *inside* the notion of a proof, which does not alter the overarching definition given at the beginning of this section.

The distinction between hypothetical and actual proofs upturns Prawitz’s distinction between closed and open arguments, with hypothetical proofs (open arguments) the norm, which in a subset of cases generate actual proofs (closed arguments). We are also liberalising the notion of proof, since it is here conceived as an activity, which may or may not produce a actual, valid, proof. This is to take seriously the idea that logic is not simply a matter of consequence and the construction of valid proofs, but rather it (also) has to do with the act of proving,

<sup>13</sup>The quotation has been altered slightly to reflect the fact that I am interested in conditional rather than knowability, but the point is theirs.

<sup>14</sup>The notion of a hypothetical proof bears obvious similarity to Girard’s [15] notion of paraproof.

reasoning and the construction of judgements. As I say, above, the standard approach to the validity of a proof, stemming from Heyting [16] through Dummett [12], focuses on proof-objects. The central feature that these views share is that a proof of a formula  $\alpha$  is a construction  $\pi$  such that  $\pi$  makes  $\alpha$  true, and that knowing a proposition is to have a constructive proof of it. In distinction, we may follow Sundholm’s [39] account of constructions, which argues for a process / product distinction *inside* constructions, regarded both as processes, or as those processes taken as objects. The idea, is that a proof-object is that which remains *posterior* to the completion of a proof-act, and the trace of a proof is what is written down as the recipe for how to construct that proof.<sup>15</sup> A proof is something that is carried out in time, which then may become an object only subsequently, and in this sense, we also follow Martin-Löf’s [22] argument that: ‘[a] proof is, not an object, but an act [...], and the act is primarily the act as it is being performed, only secondarily, and irrevocably, does it become the act that has been performed.’ The process of “proving”, then, is more like a process of reasoning that is not required to live up to objective correctness conditions on assertions, which may be thought of as *generating* proof-objects.

### 3. DYNAMIC ASSERTIONS

Whilst the proofs as acts view does suggest that there might be a place for conditional assertions alongside categorical assertions, there remain obvious problems. It is fairly clear, for example, that this kind of approach is incompatible with accounts of assertion that require constitutive norms (such as the existence of proofs), which are supposed to govern the proprieties of assertions. On such accounts, assertions are taken to be an “all or nothing” affair. There are a number of accounts of assertion that require constitutive norms on the making of assertions, such as that one must make an assertion, “c”, only in case one knows that “c”, as held by Williamson [44]; or that one must make an assertion, “c”, only in case it is true that “c”, as held by Weiner [43]; or that one must make an assertion, “c”, only in case a proof of “c” exists [8, e.g.]. These accounts all share the idea that categorical assertions are taken to be “industry standard”. So, even if conditional assertions were allowed some sort of existence, the “all or nothing” status awarded to assertions means that conditional assertions would be treated as “empty”. I will not discuss the merits, or otherwise, of these views (for this, see the excellent discussion in [19, 28, 29]).<sup>16</sup> Rather, I want to point to an alternative account of assertion that coheres much better with the approach to proofs and proving given above.

<sup>15</sup>The analogy Sundholm makes is that written proofs are like annotations for a game of chess, as opposed to proof-acts, which are like the game itself.

<sup>16</sup>Additionally, it may well be the case that these views can be made compatible with the account proposed below in some way.



In [19], this is called the COMMITMENT view, where it is traced back to the work of Peirce [30] who suggests that ‘to assert a proposition is to make oneself responsible for its truth’ (p.384).<sup>17</sup> The key distinction between this, and the constitutive norms approach is, as Macfarlane [19] puts it; ‘[...] while the constitutive rules approach looks at upstream norms - norms for *making* assertions - the commitment approach looks at downstream norms - the normative *effects* of making assertions’. This view is, perhaps, made most clear in the account of assertion games given by Robert Brandom [1, 4].<sup>18</sup> In [4], Brandom suggests that asserting that “c” is to undertake a commitment to defend “c” when challenged. So, the emphasis here is not on *prescribing* the conditions under which it is permissible to make an assertion, but rather it is an account of what is *proscribed after* an assertion has been made. So, plausibly the key norm on assertion is not a commitment to its truth, but rather a commitment to *defend* its truth. In this vein, Pagin [28] also draws attention to the relationship between assertions and promises, as discussed by Watson [42], where Watson notes their similarities. The key distinction between the two, according to Watson, is that the commitment involved is to something that is speaker-independent, which is just the defensibility, rather than the truth, of the assertion. So, again, on this view, the agent making an assertion is obliged to defend the assertion if challenged [42, p.70]. That is to say, assertoric norms should not be understood to restrict what an agent ought to assert, instead they may be thought of as constraints on how agents respond to challenge in social and dialogical contexts. Furthermore, on this view, the norms on assertions have to do with a willingness to make an attempt to justify those assertions that an agent has brought “into the game of giving and asking for reasons” [2, p.57].<sup>19</sup> Importantly, then, unlike the constitutive norms approaches to assertion, on which some sort of grounds (presumably justificatory) for making an assertion are required of agents *prior* to making that assertion, the commitment approach requires only that an agent be prepared to make an attempt to justify the assertion *subsequent* to making that assertion.

It is in this sense, then, that making an assertion may be thought of as being akin to making a move in a game of reasons. This makes it available for scrutiny, so that, when asked, then agent should attempt to justify the assertion by way of providing some sort of reasons for it:<sup>20</sup>

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<sup>17</sup>I won’t discuss the theory of truth suggested by these pragmatic approaches here, but see [3] for an exposition of Brandom’s approach.

<sup>18</sup>This also follows Wittgenstein’s suggestion that making an assertion is to make a move in a game [45, §22].

<sup>19</sup>An excellent discussion of these issues can be found in [35].

<sup>20</sup>See also the excellent discussion in [20].

In asserting a claim one not only authorizes further assertions, but commits oneself to vindicate the original claim, showing that one is entitled to make it. Failure to defend one's entitlement to an assertion voids its social significance as inferential warrant for further assertions. It is only assertions one is entitled to make that can serve to entitle others to its inferential consequences. Endorsement is empty unless the commitment can be defended.<sup>21</sup> [1, p.641]

So, on this view, making an assertion is primarily a matter of bringing that assertion into "play". At this point, the assertion is subject to norms involving a commitment to its defense, to providing reasons for it, and allowing it to be "tested" through interaction with other reasons, counterexamples and so on. Assertions do not stand alone, on Brandom's [e.g. 4, p.167] view, rather, they stand in need of reasons, and it is in the context of language "games" that we ask for, and provide reasons for, our assertions.

This is to place assertions squarely where they belong: in a social setting that involves multiple agents.<sup>22</sup> Further still, what is important in this shift is that these are dialogical norms that are explanatorily prior to constitutive norms of truth or knowledge or justification.<sup>23</sup> If we think of this in relation to the distinction between conditional and categorical assertions, the commitment view can be thought of as putting conditional assertions first, whilst allowing that categorical assertions are

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<sup>21</sup>On how this approach differs from Gricean accounts, see [1], where transmission models of communication are dispensed with in favour of an interactional model.

<sup>22</sup>Pagin, in several places [27, 28, 29, e.g.], makes an argument to the effect that the social account of assertions does not, by itself, provide sufficient conditions on the nature of assertions, whilst he accepts that it may be the case that they provide necessary conditions. The discussion in [19] provides a useful rejoinder, though, in any case I do not think that this is an issue for the view espoused here. For example, the kinds of problems usually thought to face commitment approaches involve examples where assertions are made without explicitly making statements, through nonlinguistic signs, for example. I don't think that these are problematic for the account given here, since, it seems perfectly acceptable that one might ask for reasons for such signs, thereby clarifying them, in the same way as linguistic statements. A slightly different example given by Nunberg [26], and discussed in [19], is a waitress who states that "The ham sandwich left without paying". The waitress has made an assertion, though it does not seem correct to say that she has asserted that the ham sandwich left without paying. But, whilst this may seem *prima facie* problematic for a commitment view, I agree with Macfarlane [19], that, to the contrary, this view fares very well in this respect:

[...] if we wanted to settle, for example, whether Nunberg's waitress had asserted that a *sandwich* had left, or that a *person* who ordered a sandwich had left, we might ask with (if either) of these propositions she meant to commit herself to.

<sup>23</sup>Shieh [37] puts this as follows:

To be taken as making an assertion, a speaker must acknowledge that the statement she is making is subject to assessment as correct or incorrect, by reference to what she would count as justifying it. (cited in [21])

*generated* by the dynamics of assertion games. It is not the case that a conditional assertion that never generates a categorical assertion is merely “empty”, it is just that assertion is not an all or nothing kind of affair. So, rather than thinking of the making of an assertion as expressing a fully formed propositional content, which may be thought of as true or false, we rather think of it as “playing” the statement as a kind of “token” in a game.<sup>24</sup> At this point, the statement may be treated hypothetically, and can be challenged and tested by other agents. It may, for example, be defended by the provision of reasons, and it may be contested by other reasons and counterexamples. It is also the case that, at some point during this process, the agents involved might agree that adequate justification has been provided for the original statement to be considered verified, or, indeed, that there is enough reason to think that it is false. At this point, it seems that we would be in a position to evaluate the statement as a kind of propositional content, in the usual way, but this occurs only after this interaction has occurred.

**3.1. Conditionals in interaction.** Let us connect the above account of assertions with the discussion of hypothetical and actual proofs. It is clear that we can think of an initial assertion as a hypothetical proof. But, now, whilst the idea of a hypothetical proof is somewhat idiosyncratic, it is rendered more transparent if thought of as just a “play”, or “move”, in the assertion game. This may be transformed into an actual proof by providing a proof for each assumption, and at each stage of the argument providing reasons for the statement in response to “tests”. But, now, notice that this would suggest that the the act of proving is one which is intrinsically social, rather than just individual, and that, we now have an explanatory structure for this process, which is just that the agent must respond to any tests of the initial statement, where these tests are part and parcel of the process of providing a proof of the initial statement. So, the process of constructing a proof is just our “game of giving and asking for reasons”, which concerns the hypothetical, and that which is “in process”, whilst the product which is a construction is that which is made explicit (to use Brandom’s [4] terminology) over the course of this process, to the point where the initial statement is justified. That is to say, we have provided an explanation of the BHK-style approach that takes a construction to be the construction of a justification, by means of a social account of the nature of assertions and justifications.

The above approach to assertions not only provides a way of thinking about the role of assertions socially, but it also explains *why* proofs should be taken as central to any such account. This is to situate the activity of proving squarely in

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<sup>24</sup>See also [18] for a similar approach to the relationship between assertion and proving.

the dialogical approach to logic.<sup>25</sup> In this setting, proofs are required to have clear explanatory value, which piggybacks upon the interaction between proponent and opponent:

Proponent’s job is not only to “beat Opponent”; she also seeks to persuade Opponent of the truth of the conclusion, if he has granted the truth of the premises. In fact, the goal is not only to show that the conclusion follows from the premises, but also why it does; this corresponds to the idea that deductive arguments ought to have explanatory value. In this sense, Proponent and Opponent are cooperating in a common inquiry to establish what follows from the premises, and thus to further investigate the topic in question.  
[25]

In our, more generalised setting, this is just because the utterance of an assertion brings with it a commitment to its justification, in response to requests for reasons, which is tantamount to asking for proofs of the assertion to be given. So, over the process of attempting to provide a proof, agents are making conditional assertions, which may yet become categorical. We can think of this, hypothetical register as a function of the kind discussed by Martino and Usberti. A conditional assertion is like a hypothetical proof insofar as it is a function that takes a categorical assertion (antecedent), and maps it into a categorical assertion (consequent).<sup>26</sup> Whenever no categorical assertion of the antecedent exists this function remains hypothetical insofar as there is nothing yet to map into the consequent. Whenever there is such an assertion, a categorical assertion of the consequent is made. We now have a simple explanation of this process, in terms of the distinction between the *activity* of proving, and the *object* produced (a valid proof), by way of the social, and dialogical, role of assertions.

#### 4. CONCLUSIONS

The above account has provided a way of thinking through the dynamics of a social and dialogical approach to assertions, and, in doing so, we might also

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<sup>25</sup>We should note, however, that the dialogical approach advanced here, and influenced by Brandom, takes dialogue to be a largely cooperative activity, in which we are interested in reasoning together, rather than “playing against each other” as in Lorenzen or Hintikka style games. See [20] for a similar distinction.

<sup>26</sup>This does, of course, suggest that we do not have hard and fast criteria for determining which “reasons” will be taken to be sufficient to general categorical assertions, rather, we are allowing that this sufficiency may be decided only in the space of reasons, and by those agents involved. It may be argued that a more traditional realist approach to conditionals, therefore has a leg-up on the approach advanced here, since it is capable of providing a clear and objective account of the sufficient conditions under which conditionals may be truthfully made. However, it is precisely these supposedly objective conditions that get us into trouble with conditionals in the first place, and as Dummett put it, I think this is just ‘ontological mythology’ [11, p.25].

conjecture that a dual role for denial is required, alongside assertion. If, for example, we take the notion of the “game of giving and asking for reasons” seriously, then, we have a setup that involves both *assertions* and *tests*, which, in a simplified abstraction, we may think of as a interaction between two agents. If we take it that the making of an assertion brings with it a commitment to its defence, then we also require something to defend that assertion *against*. Above, I have mentioned tests, counterexamples, and so on. These *challenges* to the initial assertion may be characterised by means of denial, insofar as denial is understood to be a basic speech-act that is both distinct from, and non-interdefinable with, assertion.<sup>27</sup> That is to say, we can interpret the account given above as a kind of dialogue structure between the roles of prover and denier, where an assertion of a statement involves a commitment to its defence, and a denial of the statement involves a commitment to its challenge. As such, we can split Brandom’s “game” into two parts: the *giving* of reasons on the part of “prover”, and the *asking* for reasons on the part of the “denier”. But, how this brief suggestion is to be developed, I leave for another paper.<sup>28</sup>

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<sup>27</sup>That we require denials, alongside assertions, also coheres with the fact that on this approach to proofs, the negation of a statement can not be categorically asserted. Whilst this is a problem for a monological approach to proof, it is grist to the mill for the account offered here, in which we can naturally introduce denial alongside assertion.

<sup>28</sup>Great thanks to Fabien Schang for incisive comments on an earlier version of this paper.

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